UCSD Economics 113 Spring 2011
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Lecture Notes, April 8, 2011

Convexity

A set of points S in R^N is said to be convex if the line segment between any two points of the set is completely included in the set.

S is <u>convex</u> if x, y $\in$ S, implies $\{z \mid z = \alpha x + (1-\alpha)y,\ 0 \le \alpha \le 1\} \subseteq S$.

S is said to be <u>strictly convex</u> if x, $y \in S,\ x \neq y,\ 0 < \alpha < 1$, implies $\alpha x + (1-\alpha)y \in$ interior S .

The notion of convexity is that a set is convex if it is connected, has no holes on the inside and no indentations on the boundary. A set is strictly convex if it is convex and has a continuous strict curvature (no flat segments) on the boundary.

Economically, this notion corresponds to "diminishing marginal utility" "diminishing marginal rate of substitution" "diminishing marginal product" .

Properties of Convex Sets

Let $C_1,\ C_2$ be convex subsets of R^N. Then:

$C_1 \cap C_2$ is convex,

$C_1 + C_2$ is convex,

$\overline{C}_1$ is convex

The unit simplex in $\mathbf{R}^N$, is

$$P = \left\{ p \mid p \in R^N, p_i \geq 0, i = 1, \ldots, N, \sum_{i=1}^{N} p_i = 1 \right\}. \tag{5.1}$$

The unit simplex is a (generalized) triangle in N-space.

Note that P is compact (closed and bounded) and convex.

Theorem 5.1 (Brouwer Fixed-Point Theorem) Let $f(\cdot)$ be a continuous function, $f : P \to P$. Then there is $x^* \in P$ so that $f(x^*) = x^*$.

The four properties assumed in the Brouwer Fixed Point Theorem — continuity of f, closedness, boundedness, and convexity of P — are all essential to the theorem. Omit any one of them and the theorem fails.

The result can be generalized from P to any compact convex set.